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# Using UniPile to fit t-z or q-z functions to load-movement records

UniPile includes five alternative methods for fitting a t-z/q-z function to observed load and movements pairs. A few recommendations are presented in the following on how to proceed with the fitting.

The most common way of fitting calculated load-movement values to measured is to start with deciding which pair of load-movement records that should be the main target of the fit. We can call this pair "the target pair of resistance and movement". Fitting the analysis to this pair is the first objective and it is achieved by adjusting input  $r_{trg}$ -resistance<sup>1</sup> (the beta-coefficient, rather, in an effective stress analysis) of the soil layers involved (the effective stress-approach is superior to the total stress approach in describing the actual soil response). If the soil profile involved consists of a single soil layer, the process is easy. Multiple soils layers make it correspondingly complex. This fit returns the target load, but not the target movement.

The second part of the fit is to by trial-and-error fit the calculated load-movement curve to the measured by selecting appropriate t-z (or q-z) curves and adjusting them to achieve a fit before and after the target point. (See Example 1, below). Note, the target pair normally comprises a certain length of the pile and the target movement includes the effect of pile compression, whereas the input to UniPile refers to the load and movement of the individual short pile elements of the soil layer addressed (per the input of "set"" for each layer. The input of the  $\delta_{trg}$ -movement ( $\delta_u$ -movement) for each function tried often needs to be a bit smaller than that measured to make a good fit to the target pair of resistance and movement.

UniPile includes five t-z and q-z functions. For each function, the User needs to input the  $\delta_{trg}$ -movement ( $\delta_u$ -movement) of the  $r_{trg}$ - $\delta_{trg}$  target pair. (The 100-% "force" is the  $r_{trg}$ -resistance and if that needs to be change, it means starting over at to square one ). For each t-z (or q-z) function tried, the function coefficient and the movement input for  $\delta_{trg}$  are varied After trying a couple of the five functions for each soil layer, the function that gave the best fit is selected to represent the best-fit analysis results, thus far, and then used for fin-tuning the fit.

It is convenient to export the measured load-movement data to an Excel spread sheet. That is, once the efforts of fitting start to return a seemingly reasonable fit, each trial should be exported to a text file that is imported to Excel and plotted so as to show in visual detail the goodness of the fit.

## 1. The Ratio Function

The Ratio Function expresses a strain-hardening pile load-movement. For a target pair of values of load,  $r_{trg}$ , and movement,  $\delta_{trg}$ , the load-movement resistance is shown in Eq. 1.

Eq. 1



<sup>&</sup>lt;sup>1</sup> UniPile refers to the target resistance and movement input as  $r_u$  and  $\delta_u$ , respectively. To avoid confusion with the various definitions of ultimate resistance—capacity—in use in the industry, they are here instead called  $r_{trg}$  and  $\delta_{trg}$  respectively—"trg" for "target".

where	r	=	force variable (shaft resistance or toe stress)
	r <sub>trg</sub>	=	target resistance
	δ	=	movement variable)
	$\delta_{trg}$	=	target movement, i.e., movement at target resistance
	θ	=	an exponent; $0 \le \Theta \le 1$

The shape of the calculated load-movement response is adjusted by means of the  $\theta$ -exponent until a best-fit is achieved. For shaft resistance, the  $\theta$ -exponent usually ranges from 0.1 through 0.6. For toe resistance, it ranges from about 0.5 through 0.8. A  $\theta$ -exponent equal to 1.0 is a straight line.

The Ratio Function is usually one that best expresses the toe load-movement response.

#### 2. The Chin-Kondner Function (Hyperbolic)

The Chin-Kondner Function (a hyperbolic function) is expressed in Eq. 2. It can be called strainhardening, although the increase of resistance with increasing movement is not particularly pronounced.

Eq. 2 
$$r = \frac{\delta}{C_1 \delta + C_2} \frac{1}{r_{trg}}$$
 where  $r_{trg} = \frac{\delta_{trg}}{C_1 \delta_{trg} + C_2}$ 

where

 $\begin{array}{lll} r &=& shaft shear force variable (or toe stress) (\%) \\ r_{trg} &=& shaft shear force for \delta_{trg} \\ \delta_{trg} &=& the target movement \\ C_1 &=& the slope of the line in a r/\delta vs. \delta diagram; the Chin-Kondner plot \\ C_2 &=& ordinate intercept the r/\delta vs. \delta diagram \end{array}$ 



The shape of the calculated load-movement response is adjusted by means of the  $C_1$ -coefficient until a best-fit is achieved. The inverse of the  $C_1$ -coefficient is the resistance at infinite movement. Input of  $C_1$ -coefficients from 0.0083 through 0.0050 will result in a range of  $r_{inf}$  from 120% through 200%. The  $C_2$ -coefficient is not necessary for adjusting the fit. It is only included in order to let the User, if so-desirering, get all factors necessary needed for to calculate and plot the Chin-Kondner hyperbolic function equation separately.

The Hyperbolic Function is often the one that best expresses shaft resistance in clay.

## **3.** The Exponential Function

The Exponential Function expresses a load-movement shape that is very close to an initial "elastic' portion transferring to a "plastic" response, i.e., "ultimate resistance". Its load-movement relation is shown in Eq. 3.

Eq. 3 
$$r = r_{inf} (1 - e^{-b\delta})$$
 where  $r_{trg} = r_{inf} (1 - e^{-b\delta_{trg}})$ 

where r = shaft shear force (toe stress) variable

- $r_{inf}$  = shaft shear force (or toe stress) at infinite movement
- $r_{trg}$  = shaft shear force for  $\delta_{trg}$
- $\delta$  = movement variable
- $\delta_{trg} \hspace{0.1 cm} = \hspace{0.1 cm} the \hspace{0.1 cm} target \hspace{0.1 cm} movement$
- e = base of the natural logarithm = 2.718
  - = coefficient

b



The User selects the coefficient "b" that makes the first 100-% point on the function curve appear at the target movement.

## 4. The Hansen 80-% Function

The Hansen 80-% Function is expressed in Eq. 4.

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Eq. 4 
$$r = \frac{\sqrt{\delta}}{C_1 \delta + C_2}$$

where

 $r_{trg}$ shaft shear force variable (or toe stress) = r movement variable δ =  $C_1$ = the slope of the straight line in the  $\sqrt{\delta}/r$  versus movement ( $\delta$ ) diagram ordinate intercept of the straight line in the  $\sqrt{\delta}/r$  versus movement ( $\delta$ ) diagram  $C_2$ = r<sub>trg</sub> = target resistance  $\delta_{trg} \ =$ target movement

and

$$r_{trg} = \frac{1}{2\sqrt{C_1C_2}} \qquad \delta_{trg} = \frac{C_2}{C_1}$$

It is not possible to change the shape of the Hansen 80-% Function without also changing the target movement,  $\delta_{trg}$ . Therefore, the Hansen 80-% Function has a limited use with regard to fitting measured load-movement unless the input of the movement,  $\delta_{trg}$ , is equal to the movement for the measured peak results in a simulated shape that is similar to that of the shape of the measured load-movement, in particular for the strain-softening part (movement beyond the target movement).

#### 4. The Zhang Function

The Zhang Function expresses a strain-softening pile load-movement. For a target pair of values of load,  $r_{trg}$ , and movement,  $\delta_{trg}$ , the relation is shown in Eq. 5.

Eq. 5 
$$r = \frac{\delta(a+c\delta)}{(a+b\delta)^2}$$

where

$$r = shaft shear force variable (or toe stress)$$
  

$$\delta = movement variable$$
  

$$r_{trg} = target (peak) resistance$$
  

$$\delta_{trg} = movement at target (peak) resistance$$
  
a, b, and c = coefficients ("b" and "c" are functions of "a")  

$$\Gamma = ratio of strain-softening r at large movement versus ru$$

The "b" and "c" coefficients depend of the "a" parameter.

$$b = \frac{1}{2r_{peak}} - \frac{a}{\delta_{peak}} \qquad c = \frac{1}{4r_{peak}} - \frac{a}{\delta_{peak}}$$

The resistance at infinite movement,  $r_{inf}$  is:  $r_{inf} = \frac{c}{r_{\mu}b^2}$ 



The shape of the Zhang Function is controlled by input of the 'a'-coefficient. The larger the 'a', the more pronounced the strain-softening after the peak. However, the  $r_{inf}$  cannot become smaller than zero, which determines the largest acceptable input of 'a' for different target movements,  $\delta_{trg}$ . Thus, for a range of target movements from 1 mm through 80 mm, the 'a'-coefficient must be smaller than listed below.

$\delta_{trg}$ (mm)	1	2	3	4	5	6	7	8	9	10
a	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0200	0.0225	0.0250
$\delta_{trg}$ (mm)	12	15	20	25	30	40	50	60	70	80
a	0.0300	0.0375	0.0500	0.0625	0.0750	0.1000	0.1250	0.1500	0.1750	0.2000

#### **EXAMPLE 1**

A 914-mm diameter, 40.5 m long bored pile was constructed in Houston, TX. The upper 5.0 m was sleeved off to eliminate any shaft resistance along that length. A bidirectional cell (BDC) was placed at 27.0 m depth. The soil profile consisted of 17 m of silty clay followed by a 10 m thick intermediate layer of clayey and silty sand on silty clay. The groundwater table was at 5.0 m depth and the pore pressure was hydrostatically distributed. A bidirectional test was performed 28 days after the pile was concreted. The load-movement results of the test are shown below.



Final fit

Measured and Equivalent head-down load-distributions

The target pairs for the analysis are indicated in the figure and the numerical values are: BDC load = 3,610 kN and  $\delta_{upward}$  14.9 mm and  $\delta_{downward}$  = 11.6 mm. In a first attempt to fit load-movement curves calculated by t-z and q-z functions, to model the shaft resistance, a Chin-Kondner function was tried in the upper clay layer, a Ratio function in the sand, and an Exponential Function in the lower clay layer, with C<sub>1</sub> and exponents of 0.0070, 0.40, and 0.30, respectively. The first try toe-function was the Ratio Function with a exponent of 0.70. The results are shown in the second figure. By trial and error, a best-fit was obtained using the Exponential Function for the upper clay layer (exponent = 0.40), the Ratio Function for the sand layer (coefficient = 0.100), and the Ratio Function (coefficient = 0.15) for shaft resistance in the clay below the BDC. The toe resistance was modeled by the Ratio Function (coefficient = 0.500). The final fit to the measured load-movement is shown in the third figure. Once the fit is achieved, UniPile calculated also the load distribution for the target load (shown in the fourth figure).

Many are interested in seeing the Equivalent head-down load-movement, i.e., the simulation of a conventional head-down static loading test, which UniPile also will also from the fitted results as shown below. The common manual calculation of the equivalent head-down curve does not consider the water force or the often larger stiffness of the soils immediately above the BDC that are first engaged in the test and opposed to the head-down test engaging them last. UniPile does include both these facts.



The equivalent head-down load-movement curves